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RN-6221

B. E. II (Sem. III) (IC) Examination

May / June – 2010

Engg. Maths - III

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	Seat No. :
Fillup strictly the details of ☛ signs on your answer book.	
Name of the Examination : ☛ **B. E. 2 (Sem. 3) (IC)**	
Name of the Subject : ☛ **Engg. Maths - 3**	
☛ Subject Code No. : **6 2 2 1** ☛ Section No. (1, 2,.....) : **1&2**	Student's Signature

(2) Attempt **all** questions.

(3) Answer **each** section **separately**.

(4) Figures on **right** indicate marks.

SECTION - I

1 (a) Do as directed : **10**

(1) What does $x^2 + y^2 = ay$ represent in space ? Show it pictorially.

(2) Write a formula to find mass of a plane lamina in polar co-ordinate system.

(3) Find grad r, where r is the magnitude of position vector.

(4) Define irrotational vector point function. Give an example of irrotational vector.

(5) Write Dirichlet's conditions for a function to have its Fourier series expansion.

(b) Attempt any **three** : **12**

(1) Change the order of integration and evaluate

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dy\, dx}{\sqrt{y^4 - a^2x^2}}.$$

(2) Evaluate $\iint r \sin\theta \, dr d\theta$ over the area of the cordioid $r = a(1+\cos\theta)$ above the initial line.

(3) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy\, dx$ by changing to polar co-ordinates.

(4) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

2 (a) Attempt any **two** : **6**

(1) Define divergent of a vector point function. Explain its physical interpretation.

(2) In what direction from (3,1,–2) is the directional derivative of $\phi = x^2 y^2 z^4$ maximum and what is its magnitude ?

(3) Find the circulation of $\bar{F}$ round the circle c, where $\bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$ and c is the circle $x^2 + y^2 = 1,\ z = 0$.

(b) Attempt any **two** : **8**

(1) Apply Green's theorem to evaluate $\oint_c [(y - \sin x)dx + \cos x\, dy]$, where c is the plane triangle enclosed by the lines $y = 0,\ x = \pi/2$ and $y = \frac{2x}{\pi}$.

(2) Apply Stoke's theorem to evaluate $\oint_c (y\,dx + z\,dy + x\,dz)$, where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

(3) Use divergence theorem to evaluate $\iint_s \bar{F} \cdot d\bar{s}$, where $\bar{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4,\ z = 0,\ z = 3$.

3 (a) Obtain the half-range sine series for e^x in $0 < x < 1$. **4**

(b) Attempt any **two** : **10**

(1) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}.$$

(2) Find the Fourier series to represent the function $f(x)$ given by

$$\begin{aligned} f(x) &= x \quad \textit{for} \quad 0 \le x \le \pi \\ &= 2\pi - x \quad \textit{for} \quad \pi \le x \le 2\pi \end{aligned}$$

(3) Find the Fourier series to represent $f(x) = x^2 - 2$ when $-2 \le x \le 2$.

SECTION - II

4 (a) Do as directed : **10**

(1) Express $\int_0^{\pi/2} \sin^p x \cos^q x \, dx$, $p > -1$, $q > -1$, in terms of Gamma function.

(2) Define the error function. Write the value of erf (∞).

(3) Define Laplace transform. Obtain Laplace transform of 1.

(4) State and prove the first shifting theorem of Laplace transform.

(5) Write necessary and sufficient conditions for $f(z)$ to be analytic.

(b) Attempt any **two** : **6**

(1) Show that $\beta(p, q) = \int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}} dy$.

(2) Evaluate $\int_0^\infty \frac{x^c}{c^x} dx$.

(3) Show that $\frac{d}{dx}\left[erf_c(ax)\right] = -\frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$.

(c) Solve any **two** : **6**

(1) $pz - qz = z^2 + (x+y)^2$

(2) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(3) $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \frac{x-y}{xy}$

5 (a) Attempt any **one** : 6

(1) A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$, the string is given a shape defined by $f(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t>0$.

(2) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0)=3\sin n\pi x,\ u(0,t)=0,\ \ u(1,t)=0$, where $0<x<1,\ t>0$.

(b) Attempt any **two** : 8

(1) Find the Laplace transform of $f(t)=\sin^3 2t$.

(2) Find the inverse Laplace transform of $\frac{s+1}{s^2+2s}$.

(3) Solve the equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t,\ x(0)=0,\ x'(0)=1.$$

6 (a) Attempt any **two** : 8

(1) Determine a,b,c,d so that the function

$f(z)=\left(x^2+axy+by^2\right)+i\left(cx^2+dxy+y^2\right)$ is analytic.

(2) Determine the region of the w-plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$.

(3) Show that the condition for transformation $w = \frac{az+b}{cz+d}$ to make the circle $|w| = 1$ correspond to a straight line in the z-plane is $|a| = |c|$.

(b) Attempt any **two** : **6**

(1) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ along the real axis to 2 and then vertically to $2+i$.

(2) Evaluate $\oint_c \frac{\cos z}{z - \pi} dz$, where c is the circle $|z - 1| = 3$.

(3) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^5} dz$, where c is the circle $|z| = 2$.